Unified Engineering Fall 2007
Signals & Systems Problems

Assigned: Friday, November 30
Due: Friday, December 7
Note: Please hand in problems on separate sheets of paper

Note: The problems marked with '*' are ones that will be useful to do as part of your Quiz preparation.

Problem S8.1: Look-back to Lectures S10, S11, S12 (10 points)

The linearized longitudinal motion of a helicopter near hover, as shown in the figure below, can be modeled by the normalized third-order system

$$\underbrace{\begin{bmatrix} \dot{q} \\ \dot{\theta} \\ \dot{v} \end{bmatrix}}_{\dot{\vec{x}}} = \underbrace{\begin{bmatrix} -0.4 & 0 & -0.01 \\ 1 & 0 & 0 \\ -1.4 & 9.8 & -0.02 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} q \\ \theta \\ v \end{bmatrix}}_{\vec{x}} + \underbrace{\begin{bmatrix} 6.3 \\ 0 \\ 9.8 \end{bmatrix}}_{B} \gamma,$$

where the input is $u(t) = \gamma(t)$, the rotor tilt angle, and the other notation is defined in the figure.

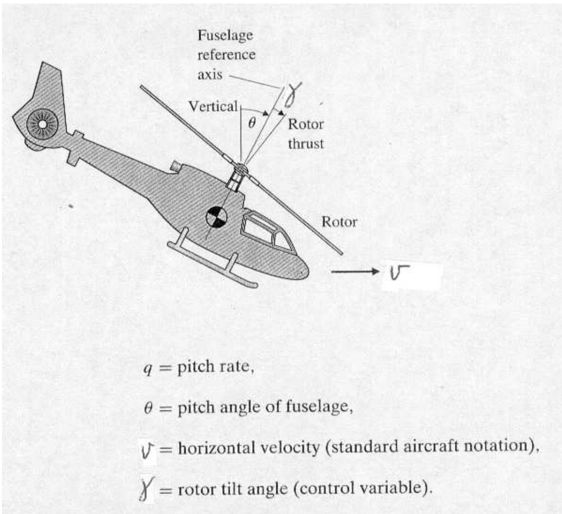

Figure 1: Sketch of helicopter hover dynamics system, Question S8.1. From Franklin, Powell and Emami-Naeini, *Feedback Control of Dynamics Systems*, Fifth edition, 2005.

*(a) Draw the block diagram for the system.

*(b) Determine, by hand, the characteristic equation of the system.

(c) Compute the eigenvalues of the system (you can use Matlab to do this).

(d) What do the eigenvalues tell you about the dynamics of the helicopter system?

(e) We wish to change the dynamics of our helicopter, using a feedback control system. If our controlled system has eigenvalues $\lambda_1 = -2$, $\lambda_2 = -1 + j$, and $\lambda_3 = -1 - j$, what should the characteristic equation of the controlled system be?

In Lectures S17 and S18, we will talk about ways to change the dynamics of a system using feedback control.

Problem S8.2: Look-back to Lecture S13 (10 points)

*(a) Prove the following properties of the state transition matrix $\Phi(t)$:

1. $\Phi(0) = I$
2. $\Phi^{-1}(t) = \Phi(-t)$
3. $\Phi(t_2 + t_1) = \Phi(t_2)\Phi(t_1)$

(b) Verify Properties 1 and 2 for:

$$\dot{\vec{x}} = \begin{bmatrix} 0 & 1 \\ -5 & -2 \end{bmatrix} \vec{x}$$

Problem S8.3: Look-back to Lectures S14, S15, S16 (10 points)

The state-space system

$$\begin{aligned} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \\ \vec{y} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \end{aligned}$$

has state transition matrix

$$\Phi(t) = \begin{bmatrix} 2e^{-t} - e^{-2t} & e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} & -e^{-t} + 2e^{-2t} \end{bmatrix}$$

*(a) Compute the output response of the system to a unit step input, $u(t) = \sigma(t)$, and initial conditions $\vec{x}(0) = [1\ 1]^T$.

*(b) Is the system underdamped, overdamped or critically damped? Explain why.

(c) Using Matlab, plot the output response computed in part (a).

Extra practice for the final: Derive the state transition matrix given above.